$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \xrightarrow{\text{anno successivo}} \begin{bmatrix} m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n \\ s_1 \cdot x_1 \\ s_2 \cdot x_2 \\ s_3 \cdot x_3 \\ \vdots \\ s_{n-1} \cdot x_{n-1} \end{bmatrix} = A \cdot x$$

popolazione di animali

$x_i = $ # animali tra $i-1$ e i anni di età

$s_i = \mathbb{P}[$animale di i anni sopravvive un anno$]$

$m_i = $ # medio di figli di un animale di i anni

$$A = \begin{bmatrix} m_1 & m_2 & m_3 & \cdots & m_{n+1} \\ s_1 & 0 & & & \\ & s_2 & \ddots & & 0 \\ & & \ddots & \ddots & \\ 0 & & & s_n & 0 \end{bmatrix} \in \mathbb{R}^{n+1 \times n+1}$$

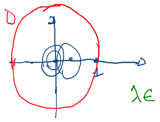
$$B = \begin{bmatrix} \frac{m_1}{\gamma} & \frac{m_2}{\gamma} & \frac{m_3}{\gamma} & \cdots & \frac{m_{n+1}}{\gamma} \\ \frac{s_1}{\gamma} & 0 & & & \\ & s_2/\gamma & \ddots & & 0 \\ & & \ddots & \ddots & \\ 0 & & & \frac{s_n}{\gamma} & 0 \end{bmatrix}$$

$m_1 + \dots + m_{n+1} = \gamma$

$s_i \leq \gamma \quad \forall i$

$[B]$ $\quad C_i : \left\{ z \in \mathbb{C} \;\middle|\; |z - b_{ii}| \leq \sum_{i \neq j} |b_{ij}| \right\}$

<u>Th</u> $\lambda \in \bigcup C_i$ se λ autovalore di B



$D = \{ z \in \mathbb{C} : |z| \leq 1 \}$

$\lambda \in \bigcup_{i=1}^{n+1} C_i \subseteq D$

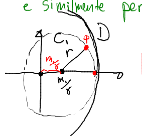
vogliamo dimostrare

Basta dimostrare che $C_i \subseteq D \quad \forall i$

$C_2 : \left\{ |z| \leq \frac{s_1}{\gamma} \right\}$

$|z| \leq \frac{s_1}{\gamma} \leq 1 \Rightarrow |z| \leq 1 \quad C_2 \subseteq D$

e similmente per $i = 3, 4, \dots n+1$



$|p| = \left| \frac{m_1}{\gamma} + y \right| \leq \left| \frac{m_1}{\gamma} \right| + |y| = \frac{m_1}{\gamma} + r$

y di modulo r

$|p| \leq \frac{m_1}{\gamma} + \frac{m_2}{\gamma} + \frac{m_3}{\gamma} + \dots \frac{m_{n+1}}{\gamma} = \frac{m_1 + \dots + m_{n+1}}{\gamma} = 1$

$C_1 \subseteq D$

$\lambda \in \bigcup_{i=1}^{n+1} C_i \subseteq D \qquad \square$

$B = \frac{1}{\gamma} A$

Se μ autovel. di A, $|\mu| \leq \gamma$

Leslie model

function $A = \text{leslie}(s, m)$

$s \in \mathbb{R}^n, \quad m \in \mathbb{R}^{n+1}$

$$\begin{bmatrix} m_1 & \cdots & & m_{n+1} \\ s_1 & 0 & & \\ & \ddots & \ddots & 0 \\ 0 & & s_n & 0 \end{bmatrix} \qquad \begin{bmatrix} \times & \times & \times & \times & \times & \times \\ \times & & & & & \\ & \times & & & & \\ & & \times & & & \\ & & & \times & \times & \end{bmatrix}$$

Metodo delle potenze

x_0 dato

$x_{k+1} = A x_k \qquad k = 0, 1, 2, \dots$

o anche

$x_k = A^k x_0$

x_k " $\longrightarrow$ " autovettore dominante di A

$$A_k x_k \approx \lambda x_k = \begin{bmatrix} \lambda (x_k)_1 \\ \lambda (x_k)_2 \\ \vdots \\ \lambda (x_k)_n \end{bmatrix} \qquad A \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\lambda \approx \frac{(A x_k)_1}{(x_k)_1}$$

$$\lambda \approx \frac{x_k^T A x_k}{x_k^T x_k}$$ quoziente di Rayleigh

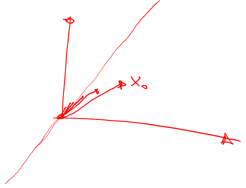
$$\frac{x_k^T \lambda x_k}{x_k^T x_k} = \lambda$$

```
x=x0
for i=1:k
    x=A*x;
end
```

$x = A^k x_0$

$A \cdot A \cdot A \cdot \dots \cdot A \qquad ((A^2)^2)^2 \qquad O(\log_2 k n^3)$

k volte $O(n^2) \sim O(kn^2) \qquad (k-1) O(n^3) \sim O(kn^3)$



se avessi $x_k = y$

$x_{k+1} = Ay = \lambda y$

$x_0 = \begin{bmatrix} 100 \\ 1000 \\ 1000 \\ 1000 \end{bmatrix}$

$x_1 = A \cdot x_0$

$x_2 = A \cdot x_1$

$\vdots$

$[3:1:1:1]$

$$\left.\begin{matrix} (x_0)_j \\ (x_1)_j \\ (x_2)_j \\ (x_3)_j \\ \vdots \\ (x_{20})_j \end{matrix}\right\} k+1$$

$v_1, v_2 \dots \quad v_{n+1}$

$x_0 = \boxed{\alpha_1} v_1 + \boxed{\alpha_2} v_2 + \dots + \alpha_{n+1} v_{n+1}$

$x_k = \alpha_1 \lambda_1^k v_1 + \alpha_2 \lambda_2^k v_2 + \dots + \alpha_{n+1} \lambda_{n+1}^k v_{n+1}$